THERMOELASTIC EFFECTS IN DISPERSE SYSTEMS

Yu. A. Buevich and A. Yu. Zubarev

UDC 536.413:620.22

We devised a continual theory of the thermoelasticity of a disperse medium consisting of a homogeneous matrix with spherical inclusions of some other material distributed in it.

If the linear scale of fields of mean temperature and deformations of a composite material or some other heterogeneous disperse system considerably exceeds the characteristic dimension of the inhomogeneities, then it is natural to describe the behavior of the system in continual approximation, introducing the effective moduli of elasticity, thermoelastic and thermophysical characteristics, etc., relating to the system as a whole. Attempts at calculating the effective coefficients of thermoelasticity known to the present authors are based either on spatial averaging [1-3] or on the use of the statistical theory of correlation moments [4]. A review of the investigations dealing with the determination of the effective properties of composite and other microinhomogeneous materials can be found, e.g., in [5, 6]. Below, in devising the theory of thermoelasticity of disperse media, we used the method of averaging over the ensemble of permissible configurations of the system of inclusions in combination with the methods of the theory of self-consistent fields developed in [7, 8]. Earlier on, these methods had been successfully used in the analysis of the rheological [8], thermophysical [9], and elastic [10] properties of disperse systems.

The advantages of ensemble averaging in comparison with spatial one were pointed out in [10]; in particular, when it is used, there is no need to require that the linear scale of the mean fields be much larger than the dimension of the representative physical volume containing a large number of discrete inclusions.

In the known works dealing with the thermoelasticity of composite materials, the authors do not take into account the heat liberation within the bulk of the phases and on their boundaries although it is bound to have a palpable effect on the effective thermoelastic characteristics similarly to the way it manifests itself in the thermophysical properties of heterogeneous media [11, 12]. Such situations arise, e.g., when a current passes through composite material, when it absorbs radiation, when phase or chemical transformations occur on phase boundaries, and also in systems containing heat-liberating elements. The method used below makes it possible to take internal heat liberation into account in a natural way.

General Relations. We consider a disperse medium with chaotically distributed spherical inclusions with radius a. We regard the materials of the matrix and of the inclusions, and also of the medium as a whole, as homogeneous and isotropic. On the surfaces of the inclusions and within the bulk of both phases heat sources with the following powers act:

$$Q_s = L^0 - L^1 T, \quad Q_j = H_j, \quad j = 0,1 \tag{1}$$

(here and henceforth the subscripts 0 and 1 denote magnitudes relating to the matrix and the inclusions, respectively). For the sake of simplicity we take it that the values of H_j, L^0, L^1 do not depend on the temperature. It is assumed that the condition of applicability of continual methods $I/a \gg 1$.

We regard the deformation of a medium having inhomogeneous temperature analogously to deformation under isothermal conditions [10]. Applying Fourier transform with respect to time, we write the equations of motion and the expressions correlating stresses with strains and temperatures in both phases in the same form

$$-\omega^2 D\mathbf{U} = \nabla\Sigma, \quad \Sigma = (K \operatorname{div}\mathbf{U} - BT)\mathbf{I} + 2M\mathbf{E}. \tag{2}$$

A. M. Gorkii Ural State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 2, pp. 248-255, August, 1986. Original article submitted June 11, 1985.

0022-0841/86/5102-0934$12.50

Here, density D, the compression modulus K, the shear modulus M, and the coefficient B are taken as generalized functions coinciding within the matrix and the inclusions with the corresponding material magnitudes [10]. Using the apparatus of ensemble averaging, we obtain from (2) equations for the mean vector of displacement **u**, the mean strain and stress tensor **e** and **σ** and the temperature τ

$$-\omega^2 d\mathbf{u} = \nabla\boldsymbol{\sigma}, \quad \boldsymbol{\sigma} = (k\varepsilon - \beta\tau - \varkappa)\mathbf{I} + 2\mu\mathbf{e}, \quad \varepsilon = \operatorname{div}\mathbf{u}, \tag{3}$$

where the effective density, the moduli k and μ, the coefficient β and the parameter ϰ, referred to the disperse medium as a whole, are determined by the relations

$$\begin{gathered} d = d_0 + (d_1 - d_0)\rho\gamma, \quad k = k_0 + (k_1 - k_0)\rho\nu_\tau, \quad \mu = \mu_0 + (\mu_1 - \mu_0)\rho\nu_e, \\ \beta = \beta_0 + (\beta_1 - \beta_0)\rho\nu_\tau - (k_1 - k_0)\rho\nu_{\varepsilon\tau}, \\ \varkappa = \varkappa_s L^0 + \varkappa_0 H_0 + \varkappa_1 H_1 = (\beta_1 - \beta_0)\rho\nu_{\tau 0} - (k_1 - k_0)\rho\nu_{\varepsilon 0}, \end{gathered} \tag{4}$$

and the parameters ν_ε, ν_e, $\nu_{\varepsilon\tau}$, $\nu_{\tau 0}$, $\nu_{\varepsilon 0}$ and γ satisfy the following formal relations (cr. [7-12]):

$$\begin{gathered} \nu_\varepsilon \varepsilon(\mathbf{R}) + \nu_{\varepsilon\tau}\tau(\mathbf{R}) + \nu_{\varepsilon 0} = \frac{1}{v}\int\limits_{r \leqslant a} \varepsilon^*(\mathbf{R}, \mathbf{r})\,d\mathbf{r}, \quad \mathbf{r} = \mathbf{R} - \mathbf{R}', \\ \nu_e \mathbf{e}(\mathbf{R}) = \frac{1}{v}\int\limits_{r \leqslant a} \mathbf{e}^*(\mathbf{R}, \mathbf{r})\,d\mathbf{r}, \quad \nu_\tau \tau(\mathbf{R}) + \nu_{\tau 0} = \frac{1}{v}\int\limits_{r \leqslant a} \tau^*(\mathbf{R}, \mathbf{r})\,d\mathbf{r}, \\ \gamma\mathbf{u}(\mathbf{R}) = \frac{1}{v}\int\limits_{r \leqslant a} \mathbf{u}^*(\mathbf{R}, \mathbf{r})\,d\mathbf{r}, \quad v = \frac{4}{3}\pi a^3. \end{gathered} \tag{5}$$

Asterisks here denote magnitudes calculated on the assumption that the point **R** lies within a particle whose center lies at the point **R'**; averaging is carried out according to the positions of the centers of the other particles. If these magnitudes are known as functions of **R**, **r**, and also of the parameters introduced in (4), then we obtain from (5) after integration a system of transcendental equations for determining the latter, and this also includes the condition of self-consistency of the submitted theory.

To determine the integrands in (5), we have to solve the elastic and thermal problem of a test particle placed in the disperse system under consideration [7-12]. The elastic problem is the natural generalization of the problem solved in [10]:

$$\begin{gathered} -\omega^2 d'\mathbf{u}'(\mathbf{r}) = \nabla\boldsymbol{\sigma}'(\mathbf{r}), \quad r > a; \quad -\omega^2 d_1\mathbf{u}^*(\mathbf{r}) = \nabla\boldsymbol{\sigma}^*(\mathbf{r}), \quad r < a; \\ \mathbf{u}', \ \varepsilon', \ \mathbf{e}' \to 0, \quad r \to \infty; \quad \mathbf{u}^*, \ \varepsilon^*, \ \mathbf{e}^* < \infty, \quad r = 0; \\ \mathbf{u}^* = \mathbf{u} + \mathbf{u}', \quad \mathbf{n}\boldsymbol{\sigma}^* = \mathbf{n}(\boldsymbol{\sigma} + \boldsymbol{\sigma}'), \quad r = a; \quad \mathbf{r} = \mathbf{R} - \mathbf{R}'. \end{gathered} \tag{6}$$

Here **u'**, ε', **e'**, and τ' have the meaning of distortions introduced by the test particle into the corresponding mean fields which in the solution of (6) are regarded as specified, the tensor σ* within the particle is correlated with ε*, **e***, τ* by an ordinary relation with the coefficients k_1, μ_1, and β_1, and the tensor σ' is determined by a relation of the type presented in (3) in which the coefficients $k' = k_0 + (k - k_0)\rho^*/\rho$, $\mu' = \mu_0 = (\mu - \mu_0)\rho^*/\rho$, $\beta' = \beta_0 + (\beta - \beta_0)\rho^*/\rho$, $\varkappa' = \varkappa(\rho^*/\rho - 1)$, $d' = d_0 + (d - d_0)\rho^*/\rho$ figure, where ρ*(r) is the bulk concentration of the inclusions near the test inclusion, dpeending ofn the special features of stacking of the inclusions, within the framework of the suggested theory it has to be regarded as a priori known.

The values of τ* and τ' figuring in (6) are found from the solution of the independent thermal problem of the test inclusion. Such a problem was dealt with, e.g., in [11, 12], and it is therefore omitted here.

In the general case the dependence ρ*(**r**) is fairly complex, and the mentioned problems of the test particles can be solved numerically only; some variants of this dependence were dealt with in [9]. However, for systems with moderate concentation of inclusions (up to $\rho \approx 0.20$-0.25) quite good results are obtained with the approximation

$$\rho^* = \rho, \tag{7}$$

corresponding to neglecting the fact that adjacent inclusions cannot overlap. Results that are approximately correct for disperse systems with high concentration can be obtained on the basis of the model

$$\rho^* = \begin{cases} 0, & a < r < 2a, \\ \rho, & r > a. \end{cases} \tag{8}$$

Concerning the problem of calculating the elastic and thermoelastic characteristics of composite materials, such models were used in the past on an empirical basis [13, 14].

Before we will assume that the forces of inertia are much smaller than the elastic forces, i.e., the left-hand sides in the equations of motion in (3) and (6) may be neglected. The conditions of feasibility of this assumption, which are closely connected with the conditions of correctness of the continual approach, were discussed in [10]. We will solve an analogous thermal problem of a test inclusion in quasisteady approximation (see [11, 12]).

Thermoelasticity in the Absence of Heat Sources. In this case the coefficients in (1) are equal to zero, and it can be shown that $\tau^* = \tau(\mathbf{R})$, $\tau' = 0$. In the approximation (7) the problem (6) reduces to the well-known problem of thermal strain of a sphere situated in an unbounded homogeneous medium whose properties coincide with the effective properties of the disperse system. The solution of this problem has the form

$$\mathbf{u}^* = A\mathbf{r}, \quad \mathbf{u}' = B\mathbf{r}/r^3,$$

where the constants of integration are determined from the obvious boundary conditions on the surface of the sphere $r = a$. From the respective equations in (4) and (5) after simple calculation we have $\varkappa = 0$ and

$$\beta = \left(1 - \rho\frac{k_1 - k_0}{k_1 + 4\mu/3}\right)^{-1}\left\{\beta_0 + \rho\left[\beta_1 - \beta_0 - \beta_1\frac{k_1 - k_0}{k_1 + 4\mu/3}\right]\right\}. \tag{9}$$

In the approximation (8) the solution of (6) is written as follows:

$$\mathbf{u}^* = A\mathbf{r}; \quad \mathbf{u}' = C\mathbf{r} + B\mathbf{r}/r^3, \quad r < 2a; \quad \mathbf{u}' = D\mathbf{r}/r^3, \quad r > 2a,$$

where the constants of integration are calculated form the conditions of continuity of the vector of displacement and density of the force on the spheres $r = a$ and $r = 2a$. In this case again, $\varkappa = 0$, and instead of (9) we obtain

$$\beta = \left(1 - \rho\frac{k_1 - k_0}{\Gamma}\right)^{-1}\left\{\beta_0 + \rho\left[\beta_1 - \beta_0 - \beta_1\frac{k_1 - k_0}{\Gamma}\right]\right\}, \tag{10}$$

$$\Gamma = k_1 + (4/3)\,[8\,(3k_0 + 4\mu)\,\mu_0 - 3\,(\mu - \mu_0)\,k_0]\,[8\,(3k_0 + 4\mu) + 4\,(\mu - \mu_0)]^{-1}.$$

The effective modulus of hydrodynamic compression k and the effective shear modulus μ of the system were dealt with in [10]. From (9) and (10) there follow the expressions for the effective coefficient β/k of thermal expansion of the disperse system. For a dilute system in linear approximation with respect to ρ we obtain from (9) or (10) the formula

$$\beta = \beta_0 + (\beta_1 - \beta_0)\left(1 - \frac{k_1 - k_0}{k_1 + 4\mu_0/3}\right)\rho, \tag{11}$$

which coincides with the formula ensuing from [2].

If even one of the phases of the system is a viscoelastic body or an elastoviscous liquid, then k and μ depend on the frequency, regardless of whether the problems of the test particle are solved in quasisteady approximation [10]. This has to do with the fact that for viscoelastic bodies, e.g., $k_j \to k_j + i\omega\xi_j$ and $\mu_j \to \mu_j + i\omega\eta_j$, where η_j and ξ_j are the coefficients of shear and bulk viscosity. It is obvious that in this case, too, β depends on the frequency, i.e., in thermoelastic deformation of the disperse system relaxation phenomena are of importance. In this connection we will deal more in detail with dilute disperse systems of which one phase is an ideally elastic body and the other a compressible Newtonian liquid.

Using (11) and the results of [10], we come to the conclusion that for the spherical part σ_p of the stress tensor the following equality is correct:

$$\left(k_1+\frac{4}{3}\mu_0\right)\sigma_p=\left[k_0\left(k_1+\frac{4}{3}\mu_0\right)+(k_1-k_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]\varepsilon-$$
$$-\left[\beta_0\left(k_1+\frac{4}{3}\mu_0\right)+(\beta_1-\beta_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]\tau. \tag{12}$$

On the basis of the "elastic-viscoelastic" analogy [6], which is correct when the nonsteady terms in the boundary-value problems of the test particle are neglected, and after inverse Fourier transformation for an elastic matrix with spherical cavities filled with a viscous liquid, we obtain from (12):

$$\left(k_1+\frac{4}{3}\mu_0+\xi_1\frac{\partial}{\partial t}\right)\sigma_p=\left[k_0\left(k_1+\frac{4}{3}\mu_0\right)+\right.$$
$$\left.+(k_1-k_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]\left(1+T_\varepsilon\frac{\partial}{\partial t}\right)\varepsilon-\left[\beta_0\left(k_1+\frac{4}{3}\mu_0\right)+\right.$$
$$\left.+(\beta_1-\beta_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]\left(1+T_\tau\frac{\partial}{\partial t}\right)\tau,$$
$$T_\varepsilon=\left[k_0+\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]\left[k_0\left(k_1+\frac{4}{3}\mu_0\right)+\right. \tag{13}$$
$$\left.+(k_1-k_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]^{-1}\xi_1,$$
$$T_\tau=\beta_0\left[\beta_0\left(k_1+\frac{4}{3}\mu_0\right)+(\beta_1-\beta_0)\left(k_0+\frac{4}{3}\mu_0\right)\rho\right]^{-1}\xi_1.$$

Analogously for the suspension of elastic spheres in a viscous liquid we have

$$\left(1+\frac{4}{3}\frac{\eta_0}{k_1}\frac{\partial}{\partial t}\right)\sigma_p=k\left(1+T_\varepsilon'\frac{\partial}{\partial t}+T_\varepsilon''\frac{\partial^2}{\partial t^2}\right)\varepsilon-\beta\left(1+T_\tau'\frac{\partial}{\partial t}\right)\tau,$$
$$T_\varepsilon'=\left\{\xi_0k_1+\frac{4}{3}\eta_0k_0+\left[(k_1-k_0)\left(\xi_0+\frac{4}{3}\eta_0\right)-\xi_0k_0\right)\rho\right\}(kk_1)^{-1},$$
$$T_\varepsilon''=\xi_0\left[\frac{4}{3}\eta_0-\left(\xi_0+\frac{4}{3}\eta_0\right)\rho\right](kk_1)^{-1}, \tag{14}$$
$$T_\tau'=\left[\frac{4}{3}\beta_0\eta_0+(\beta_1-\beta_0)\left(\xi_0+\frac{4}{3}\eta_0\right)\rho\right](\beta k_1)^{-1},$$
$$k=k_0+(k_0/k_1)(k_1-k_0)\rho,\quad \beta=\beta_0+(k_0/k_1)(\beta_1-\beta_0)\rho.$$

It can be seen from (13) and (14) in particular that the thermoelastic effects do not influence the time of relaxation but they determine the time of aftereffect. An analogous conclusion may also be applied to systems with higher concentration.

Thermoelasticity in the Case of Spatial Sources. Let $H_j \neq 0$ but $L^0 = L^1 = 0$. Such a situation arises, e.g., when an electric current passes through composite material, in systems containing heat-liberating elements, etc. For the sake of simplification we will deal with moderately concentrated media for which (7) is approximately correct. Analogously to what was done in [11, 12], it is expedient to expand the mean temperature $\tau(\mathbf{R})$ and the displacement $\mathbf{u}(\mathbf{R})$ in the vicinity of the point $\mathbf{R}'$ into Taylor series, and then to transform such expansions into series in terms of Legendre functions. To obtain the required results it suffices to consider the case when all the mean values depend solely on one coordinate which we denote $z = r\cos\theta$, θ is the polar angle of the system of spherical coordinates. According to consideration completely analogous to those in [11, 12] it is possible, by solving the steady-state thermal problem of the test particle, to find the temperature τ^* inside this particle and the perturbation of the temperature τ' outside it. Then, solving the boundary-value problem (6), we obtain expressions for u_r^* and u_r' in the form of series in terms of Legendre polynomials, and for u_θ^* and u_θ' we obtain series in terms of the derivatives of these polynomials with respect to θ. After that we have to express the mean values figuring in these expressions and determined at the point $\mathbf{R}'$ through analogous magnitudes in $\mathbf{R}$, calculate the integrands in (5) and integrate, and then solve the equations obtained for effective coefficients characterizing the deformation of the disperse medium. Omitting all the intermediate, extremely cumbersome calculations, we will show that as a result hence follow relations (9) for β, an equation for k coinciding with the one obtained in [10], and a formula for $\varkappa$ in (3) which is also very cumbersome. We therefore write it for the special case only, when $\rho \ll 1$, and heat sources are solely inside the inclusions of the disperse phase:

$$\varkappa = \rho H_1 \frac{a^2}{\lambda_1} \left\{ (\beta_1 - \beta_0) \left(\frac{1}{2} + \frac{\lambda_1}{\lambda_0} \right) - \frac{k_1 - k_0}{k_1 + 4\mu_0/3} \left[(\beta_1 - \beta_0) \frac{\lambda_1}{\lambda_0} + \frac{\beta_1}{10} \frac{5k_1 + 4\mu_1 - (4/3)(\mu_1 - \mu_0)}{k_1 + 4\mu_0/3} \right] \right\}. \quad (15)$$

Thus we arrive at the fundamentally important conclusion that when the disperse material contains distributed heat sources, an additional spherical component appears in the tensor of effective mean stresses; this describes the origin of an additional positive or negative pressure. formally this is connected with the fact that the difference of the mean phase temperature even under steady conditions ceases to be proportional to τ, and physically it means that thermal expansion of any phase leads to the appearance of deformations, and consequently to corresponding stresses in the other phase. As a result there arises an additional mean pressure not "compensated" by deformation; its sign depends on the relations between the elastic, thermoelastic, and thermophysical properties of the materials of the phases.

Thermoelasticity in the Presence of Surface Effects. We now adopt $H_j = 0$, but $L^0 \neq 0$, $L^1 \neq 0$ in (1). An analysis of such a situation is carried out completely analogously to the preceding one, but to obtain the solution of the thermal problem of the test particle we may use directly the results of [11]. Omitting again the cumbersome calculations, we present the results for dilute disperse media. The expression for k coincides again with the expression obtained in [10], and for β and $\varkappa$ we have

$$\begin{gathered} \beta = \beta_0 + (\beta_1 - \beta_0) \left(1 + \frac{aL^1}{\lambda_0} \right)^{-1} \left(1 - \frac{k_1 - k_0}{k_1 + 4\mu_0/3} \right) \rho, \\ \varkappa = (\beta_1 - \beta_0) \frac{aL^0}{\lambda_0} \left(1 + \frac{aL^1}{\lambda_0} \right)^{-1} \left(1 - \frac{k_1 - k_0}{k_1 + 4\mu_0/3} \right) \rho. \end{gathered} \quad (16)$$

Thus the surface heat sources lead, first of all, to the same effect as bulk sources. In addition it turns out that the temperature dependence of the power of the sources affects both β and $\varkappa$.

The demonstrated method can also be applied successfully in the analysis of the influence of surface effects of another physical nature on the thermoelastic properties of disperse media. Here we will actually examine the influence of surface tension which is important in the case of foams and finely disperse emulsions. In this case the expression for the local stress tensor in (2) has to be rewritten in the form

$$\boldsymbol{\Sigma} = [K \operatorname{div} \mathbf{U} - BT + \alpha (R_1^{-1} + R_2^{-1}) \Phi(\mathbf{R})] \mathbf{I} + 2M\mathbf{E}, \quad (17)$$

where α is the surface tension on the particle surfaces, R_1 and R_2 are their principal radii of curvature,

$$\Phi(\mathbf{R}) = \sum_j \delta(\mathbf{R} - \mathbf{R}_j), \quad \mathbf{R}_j \in S_j,$$

where S_j is the designation of the surface of the j-th particle, $\delta(x)$ is a delta function. When we average (17) by the standard method with respect to the positions of all particles, we obtain the previous expression for σ in (3), and instead of the relation ensuring from (4), (5) we have

$$k\varepsilon - \beta\tau - \varkappa = k_0\varepsilon - \beta_0\tau + (k_1 - k_0)\frac{\rho}{v} \int\limits_{r \leqslant a} \varepsilon^*(\mathbf{R}, \mathbf{r})\, d\mathbf{r} - (\beta_1 - \beta_0)\frac{\rho}{v} \int\limits_{r \leqslant a} \tau^*(\mathbf{R}, \mathbf{r})\, d\mathbf{r} + \alpha \frac{\rho}{v} \int\limits_{r \leqslant a} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) d\mathbf{r}. \quad (18)$$

The elastic problem of the test particle is formulated completely analogously to (6) with a view to the surface pressure contained in the condition of continuity of the surface force. Assuming that there are not heat sources, and solving the thermal and elastic problem of the test particle by taking into account that there are no steady-state shear stresses in liquids and gases, we obtain after some calculations in the approximation (7) of a moderately concentrated medium the following formula for β:

$$\beta=\left(1+\frac{k_1-k_0+2\alpha/a}{k_1-2\alpha/3a}\right)^{-1}\left[\beta_0(1-\rho)+\beta_1\rho\left(1-\frac{k_1-k_0+2\alpha/a}{k_1-2\alpha/3a}\right)\right], \tag{19}$$

which agrees with (9) when $\alpha = 0$, $\mu = 0$. In the approximation (8), which is suitable for concentrated systems, exactly the same kind of formula is obtained. In both cases $\varkappa = 0$.

The effective coefficient of thermal expansion of a disperse medium can be determined as β/k, i.e., the formula for it follows directly from the relations for k and β.

The magnitudes k and μ, dealt with in [10] and in the present article, are the effective isothermal moduli of hydrostatic pressure and shear of the medium. Knowing the effective coefficient of thermal expansion enables us to find by the ordinary rules [15] the corresponding adiabatic moduli which are important, e.g., in the analysis of the propagation of acoustic waves in a composite material.

In conclusion, we note that generalization of the theory to nonsteady heating and deformation regimes does not pose any fundamental difficulties but it entails calculations that are even more cumbersome. If the true nature of the stacking of adjacent particles near any isolated one is to be taken into account more rigorously, the corresponding thermal and elastic problems of the test particle have to be solved numerically.

NOTATION

a, particle radius; B, local coefficient of thermoelasticity; D, local density, d, mean density; **E, e,** local and mean strain rate tensor, respectively; H, density of the bulk heat sources; **I,** unit tensor; K, k, local and effective modulus of hydrostatic compression, respectively; L^0, L^1, parameters determining the power of surface heat sources; M, local shear modulus; **n,** unit vector of the normal; Q, power of the heat sources; **R, R',** radius vectors; **r = R − R'**; T, local temperature; T_ε, T_τ, times of after-effect; t, time; **U, u,** local and mean displacement vector, respectively; v, volume of the particle; α, surface tension; β, effective thermoelastic coefficient; Γ, γ, parameters introduced in (10) and (4), respectively; ε, div **u**; η, shear viscosity; $\varkappa$, additional thermoelastic pressure; λ, thermal conductivity; μ, effective shear modulus; ν, parameters in (4) and (5); ξ, bulk viscosity; ρ, ρ^*, mean and nominal bulk concentration, respectively; $\boldsymbol{\Sigma}$, $\boldsymbol{\sigma}$, local and mean stress tensor, respectively; $\sigma_p\mathbf{I}$, spherical part of $\boldsymbol{\sigma}$; τ, mean temperature; ω, frequency; the subscripts 0 and 1 pertain to the dispersing and the dispersed phases, respectively, an asterisk as superscript to magnitudes determined inside the test particle.

LITERATURE CITED

1. B. W. Rosen and Z. Hashin, "Effective thermal expansion coefficient and specific heat of composite materials," Int. J. Eng. Sci., 8, No. 2, 157-162 (1970).
2. V. M. Levin, "Determination of the elastic and thermoelastic constants of composite materials," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 6, 137-145 (1976).
3. V. V. Novikov, "The effective coefficient of thermal expansion of inhomogeneous material," Inzh.-Fiz. Zh., 44, No. 6, 969-977 (1983).
4. P. L. Khoroshun and N. S. Saltanov, The Thermoelasticity of Binary Mixtures [in Russian], Naukova Dumka, Kiev (1984).
5. T. D. Shermergor, The Theory of Elasticity of Microinhomogeneous Media [in Russian], Nauka, Moscow (1977).
6. R. Christensen, Introduction to the Mechanics of Composites [Russian translation], Mir, Moscow (1982).
7. Yu. A. Buevich, Yu. A. Korneev, and I. N. Shchelchkova, "Heat or mass transfer in disperse flows," Inzh.-Fiz. Zh., 30, No. 6, 979-985 (1976).
8. Yu. A. Buevich and I. N. Shchelchkova, "Flow of dense suspensions," Progr. Aerospace Sci., 18, No. 2, 121-150 (1978).
9. B. S. Endler, "Effective viscosity and thermal conductivity of a disperse medium," Inzh.-Fiz. Zh., 37, No. 1, 110-117 (1979).
10. Yu. A. Buevich and A. Yu. Zubarev, "The effective rheological properties of a disperse mixture of viscoelastic materials," Inzh.-Fiz. Zh., 50, No. 3, 434-445 (1985).
11. Yu. A. Buevich, "The effect of phase transition on heat and mass transfer in disperse flows," Inzh.-Fiz. Zh., 32, No. 4, 625-631 (1977).
12. A. Yu. Zubarev, "The effect of internal heat sources on the heat transfer in disperse systems," Inzh.-Fiz. Zh., 50, No. 2, 272-278 (1986).

13. E. H. Kerner, "The elastic and thermoelastic properties of composite media," Proc. Phys. Soc., B69, No. 9, 808-820 (1956).
14. R. Hill, "A self-consistent mechanics of composite materials," J. Mech. Phys. Solids, 13, No. 2, 213-221 (1965).
15. L. D. Landau and E. M. Lifshits, Theory of Elasticity [in Russian], Nauka, Moscow (1965).

METHOD OF CALCULATING THE THERMAL CONDUCTIVITY OF POROUS GRANULAR MATERIALS WITH METAL FILLER IN DIFFERENT MEDIA

G. N. Dul'nev, B. L. Muratova,
T. V. Tribel', Kh. Madzhidov,
and M. M. Safarov

UDC 536.2

The article suggests a method of calculating the thermal conductivity of porous granular materials with metallic filler. The results of the calculation are compared with the experimental data. The error of the calculation is commensurable with the error of specifying the initial data.

To intensify technological processes at high temperatures, porous granulated aluminum oxide with metallic filler is used as catalyst. The aluminum oxide is made in the form of granules with 0.8-1.25-mm diameter. In dependence on the temperature, the weight concentration of the metallic particles n_{met}, and the composition of the gaseous medium, the thermophysical properties of the material change, and this affects the conditions of its operation.

Investigation of heat transfer in charges of porous aluminum oxide with metallic filler enable us to evaluate the range of change of thermophysical properties in dependences on the above parameters.

We will carry out the analysis on a model in the form of a charge of grains (spheres) with the same diameter; the grains themselves have a porous structure, and in the walls of the pores and in its surface metal particles are embedded (Fig. 1).

We will consider the processes of heat transfer through such a structure stage by stage. At the first stage we evaluate the thermal conductivity of a porous grain of aluminum oxide without metal particles, assuming that the pores contain gas. We use the well-known model of cracked material whose components form an interpenetrating grid [1]. The thermal conductivity of cracked material is determined by the formula

$$\lambda' = \lambda_{\mathrm{Al_2O_3}}\,[c^2M + \nu(1-c)^2 + 2\nu c\ \ (1-c)/(\nu c + 1 - c)],\quad \nu = \lambda_{\text{por}}/\lambda_{\mathrm{Al_2O_3}}, \tag{1}$$

where the parameter c is correlated with bulk porosity by the dependence $c = 0.5 - \arccos(1 - 2m_2)$ for $m_2 < 0.5$; M is a parameter characterizing the cracked state of the material.

The thermal conductivity of the pores is determined by the radiant and molecular heat transfer, and it is equal to

$$\lambda_{\text{por}} = \lambda_{\text{r}} + \lambda_{\text{m}}. \tag{2}$$

The radiant component of thermal conductivity is evaluated by the formula for radiant heat exchange between two parallel plates whose degree of blackness is ε, and the distance between them is δ (pore size) [1]:

$$\lambda_{\text{r}} = \sigma_0 (T/100)^3 \varepsilon_{\text{cd}} \delta,\quad \varepsilon_{\text{cd}} = \varepsilon/(2-\varepsilon). \tag{3}$$

The molecular component of thermal conductivity depends on many parameters, and it is determined by the formula [1]

$$\lambda_{\text{m}} = \frac{\lambda_{\text{g}}}{1 + B/(H\delta)},\quad B = \frac{4(c_p/c_v)}{1 + (c_p/c_v)}\,\frac{2-a}{a}\,\Lambda H_0 \mathrm{Pr}^{-1},\quad H_0 = 1\cdot 10^5\ \text{Pa}. \tag{4}$$

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 2, pp. 255-259, August, 1986. Original article submitted April 9, 1985.

0022-0841/86/5102-0940$12.50